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Monopoles contra vortices in SU(2) lattice gauge theory?*

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We show that the scenario of vortex induced confinement of center–projected SU(2) lattice gauge theory is not necessarily in conflict with the findings in the positive plaquette model.

Recently [1], it was reported that the vortices of center projected SU(2) lattice gauge theory reproduce the full string tension, whereas the lattice configurations fail to yield a non-zero string tension, if the vortices are suppressed. The authors argue that these vortices are the relevant degrees of freedom to confine quarks in the fundamental representation.

In order to be more precise, center–projection was defined in [1] on top of Abelian projection. The maximal Abelian gauge [2] makes a link variable $U_\mu(x)$ as diagonal as possible, and Abelian projection replaces a link variable

$$U_\mu(x) \; = \; \alpha_0(x) \; + \; i\vec{\alpha}(x) \, \hat{n}(x)\vec{\tau} \, , \qquad \alpha_0^2 + \vec{\alpha}^2 = 1 \tag{1}$$

by the Abelian link variable

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$$A \;=\; \frac{\alpha_0(x) \;+\; i\alpha_3(x)\,\tau^3}{\sqrt{\alpha_0^2+\alpha_3^2}} \;=\; \cos\theta(x) \;+\; i\sin\theta(x)\,\tau^3 \; . \tag{2}$$

Center–projection is then defined by assigning to each link variable a value ± 1 according the rule $A(x) \to \text{sign}\ (\cos\theta(x))$. A plaquette is defined to be part of the vortex, if the product of the center–projected links is -1.

This result of ref. [1] seems to be in conflict with the findings of ref. [3,4], where lattice calculations have been performed in the positive plaquette model (PPM) in the maximal Abelian gauge. In the PPM, configurations giving rise to plaquettes with negative trace are rejected. Nevertheless, this model reproduces the linear rising confinement potential. From this observation, the authors of ref. [4] concluded that vortices are not responsible for the string tension in contradiction to the findings of [1].

Here we show that the results of the vortex-approach [1] and the results of the PPM [4] are not necessarily in conflict. For this purpose, let us assume that Abelian projection has been performed and consider a particular plaquette configuration $P = U_1 U_2 U_3 U_4$, which is generated by the Abelian link variables

$$U_k \;=\; \cos\phi_k \;+\; i\sin\phi_k\,\tau^3 \; , \qquad\quad k = 1\ldots 4 \; . \tag{3}$$

The plaquette can be written as

$$P \;=\; \cos\left(\sum_{k=1}^{4}\phi_k\right) \;+\; i\sin\left(\sum_{k=1}^{4}\phi_k\right)\,\tau^3 \; . \tag{4}$$

It is easy to choose a set of link variables such that

$$\phi_1 \in [0, \frac{\pi}{2}[\; , \qquad \phi_{k=2\ldots 4} \in [\frac{\pi}{2}, \pi[\; , \qquad \sum_{k=1}^{4}\phi_k \in [0, \frac{\pi}{2}[\mod 2\pi \; . \tag{5}$$

In these cases, the trace of the plaquette P is positive. Hence such a configuration contributes in the PPM. On the other hand, center projection assigns to the link U_1

the value +1 and to the links $U_{k=2...4}$ the value -1 implying that the configuration represents a vortex, since the product of the center–projected links yields -1. Our example shows that the vortex configurations discussed in [1] are not excluded in the PPM. This result is obviously not restricted to Abelian projected links.

In order to further clarify the role of the vortices occurring in the center–projected lattice theory, we suggest to study the center-projection of the positive plaquette model.

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